

11-6
Recursion and Special Sequences

ex:

1, 1, 2, 3, 5, 8, 13, ...

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21,

$$a_n = a_{n-1} + a_{n-2}$$

Formula:

$$a_n = a_{n-1} + a_{n-2}$$

Fibonacci

ex:

$$a_{n+1} = 2a_n + 7 \quad n \geq 1$$

If $a_1 = 5$, list the first 5 terms.

$$\begin{aligned} a_1 &= 5 \\ a_2 &= 2 \cdot 5 + 7 = 17 \\ a_3 &= 2 \cdot 17 + 7 = 41 \\ a_4 &= 89 \\ a_5 &= 185 \end{aligned}$$

ex:

$$a_{n+1} = -3a_n + 2n \quad n \geq 1$$

If $a_1 = -5$, list the first 4 terms.

$$\begin{aligned} a_1 &= -5 \\ n=1 \quad a_2 &= -3(-5) + 2(1) = 17 \\ n=2 \quad a_3 &= -3(17) + 2(2) = -47 \\ a_4 &= 147 \end{aligned}$$

Example 2 Find and Use a Recursive Formula

GARDENING Mr. Yazaki discovered that there were 225 dandelions in his garden on the first Saturday of spring. He had time to pull out 100, but by the next Saturday, there were twice as many as he had left. Each Saturday in spring, he removed 100 dandelions, only to find that the number of remaining dandelions had doubled by the following Saturday.

- a. Write a recursive formula for the number of dandelions Mr. Yazaki finds in his garden each Saturday.

$$a_n = 225$$

$$a_{n+1} = (a_n - 100)2$$

- b. Find the number of dandelions Mr. Yazaki would find on the fifth Saturday.

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Ex:

Dr. Elliot is growing cells in lab dishes. She starts with 108 cells on Monday morning and then removes 20 for her experiment. By Tuesday, the remaining cells have multiplied by 1.5. She again removes 20. This pattern repeats each day in the week.

- a. Write a recursive formula for the number of cells she finds each day before she removes any.
b. How many cells will she find on Friday morning?

$$a_{n+1} = (a_n - 20) \frac{3}{2}$$

303 cells

Iteration:

Composing a function with itself repeatedly.

$f(x)$

$f(f(x))$

$f(f(f(x)))$

$$\text{ex: } f(x) = 3x - 1$$

$$x_0 = 5$$

Find the 1st, 2nd, and 3rd iterates.

$$\begin{aligned} x_1 &= 3(5) - 1 = 14 \\ x_2 &= 3(14) - 1 = 41 \\ x_3 &= 3(41) - 1 = 122 \end{aligned}$$

Find the 1st, 2nd, and 3rd iterates.

$$\text{ex: } f(x) = 4x + 3$$

$$x_0 = -2$$

$$\begin{aligned} x_1 &= -5 \\ x_2 &= -17 \\ x_3 &= -65 \end{aligned}$$

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13-23odd, 28, 29, 31-37odd